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A NOTE ON KAEHLER-EINSTEIN SPACE

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Abstract- The purpose of the paper is to obtain some of the properties of Kaehlerian manifolds of W-curvature tensor and some of those of U-curvature tensor with Kaehler-Einstein space.

INTRODUCTION:

Recently Author studied the W-curvature tensor [3],[4],[5],[6] and U-curvature tensor [3],[4],[5],[6]. Many properties of W-curvature tensor and U-curvature tensor of Kaehlerian manifold are obtained. This paper contains some important results on these tensors of Kaehlerian manifolds.

PRELIMINARIES:

Let M be a Kaehlerian manifold of complex dimension n (>1) .The complex structure tensor F_i^h, the Hermitian metric tensor g_{ji}, curvature tensor K_{kjih}, Ricci Curvature K_{ji} and scalar curvature K satisfying (see [2],[3],[4],[7])

$$F_i^h F_j^i = \delta_j^h, \nabla_j F_{ih} = 0, g_{ji} - F_j^t F_i^s g_{ts} = 0, K_{ji} - F_j^t F_i^s K_{ts} = 0,$$

where $F_{ji} = F_j^t g_{ti}, K_{kjih} = K_{kji}{}^t g_{th}$.

The Kaehlerian manifold M has the constant holomorphic sectional curvature k [7] if

$$K_{kjih} = \frac{k}{4}(g_{kh}g_{ji} - g_{jh}g_{ki} + F_{kh}F_{ji} - F_{jh}F_{ki} - 2F_{kj}F_{ih}) \ .$$

The Kaehlerian manifold M is said to be projectively flat ([1],[4]) if

$$K_{kjih} = \frac{1}{2(n+1)}(g_{kh}K_{ji} - g_{jh}K_{ki} + F_{kh}H_{ji} - F_{jh}H_{ki} - 2H_{kj}F_{ih})$$

where $H_{ji} = -K_{jt}F_i^t$.

A deviation tensor G [7] of type (0,2) with covariant components G_{ji} on M is given by

$$G_{ji} = K_{ji} - \frac{K}{2n}g_{ji} \ .$$

If $G_{ji} = 0$, then M is Kaehler-Einstein manifold and K is constant provided n>1 .

A tensor Z [7] of type (0,4) with covariant components Z_{kjih} on M is given by

$$Z_{kjih} = K_{kjih} - \frac{K}{4(n+1)}(g_{kh}g_{ji} - g_{jh}g_{ki} + F_{kh}F_{ji} - F_{jh}F_{ki} - 2F_{kj}F_{ih})$$

and it is called the H-concircular curvature tensor . If $Z_{kjih} = 0$, then M is of the constant holomorphic sectional curvature $\frac{K}{n(n+1)}$, provided n>1.

The H-projective curvature tensor P [7] of type (0,4) with covariant components P_{kjih} on M is defined by [1]

$$P_{kjih} = K_{kjih} - \frac{1}{2(n+1)}(g_{kh}K_{ji} - g_{jh}K_{ki} + F_{kh}H_{ji} - F_{jh}H_{ki} - 2H_{kj}F_{ih}) \ .$$

If

$$P_{kjih} = 0,$$

then M is projectively flat.

Recently, W-curvature tensor of type (0,4) with covariant components W_{kjih} [3],[4],[5],[6] on M has been introduced and is defined by

$$W_{kjih} = aZ_{kjih} + b_1(G_{ji}g_{kh} - G_{ki}g_{jh}) + b_2(G_{kh}g_{ji} - G_{jh}g_{ki}) \ ,$$

where a, b_1 and b_2 are some real constants.

U-curvature tensor of type (0,4) with covariant components U_{kjih} [3],[4],[5],[6] on M is defined as

$$U_{kjih} = cP_{kjih} + d_1(G_{ji}g_{kh} - G_{ki}g_{jh}) + d_2(G_{kh}g_{ji} - G_{jh}g_{ki}),$$

where c, d_1 and d_2 are some real constants.

MAIN RESULT:

Theorem: If a M is Kaehler-Einstein manifold and K is constant provided n>1 then W-curvature tensor coincide with scalar multiple of H-concircular curvature tensor, provided $a \neq 0$.

Remark: If a M is Kaehler-Einstein manifold and K is constant provided n>1 then W-curvature tensor coincide with H-concircular curvature tensor, provided $a \neq 1$.

Theorem: If a M is Kaehler-Einstein manifold and K is constant provided n>1 then U-curvature tensor coincide with scalar multiple of H-projective curvature tensor, provided $c \neq 0$.

Remark: If a M is Kaehler-Einstein manifold and K is constant provided n>1 then U-curvature tensor coincide with H-projective curvature tensor, provided $c \neq 1$.

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References:

[1] I.Hasegawa: H-projective-recurrent Kaehlerian manifolds and Bochner-recurrent Kaehlerian manifolds, *Hokkaido Math. J.* 3 (1974), 271-278.

[2] Y. Kubo: Kaehlerian manifolds with vanishing Bochner curvature tensor, *Kodai Math. Sem. Rep.* 28 (1976), 85-89.

[3] S.S.Pujar and S.G Purane: Integral formulas and inequalities in Kaehlerian manifolds and their applications-II, *Ultra Science*, 22 (2010) No.1, 205-212.

[4] S.S.Pujar and S.G Purane: Isometry of Kaehlerian manifolds with constant scalar curvature admitting a H-projective vector fields, *Bull.Cal.Math.Soc,* 102(4) p.p319-332(2010).

[5] Pujar, S.S. and Purane, S.G., Kaehlerian manifolds with vanishing W-curvature and U-curvature tensors, *Antarctica J. Math.,* 8(4) (2011), 325-334.

[6] S.G Purane : Kaehlerian manifolds admitting a metric semi-symmetric F-connection-II *Recent Research in Science and Technology*, 4(9) (2012), 16-18.

[7] K.Yano and S.Ishihara: Kaehlerian manifolds with constant scalar curvature whose Bochner curvature tensor vanishes, *Hokkaido Math. J.* 3 (1974), 294-304.

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