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PROOF OF THE TWIN PRIME CONJECTURE

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Abstaract – The twin prime conjecture (the conjecture stating that there are infinte number of twin prime pairs) has been proved in his manuscript using basic congruences and representing the equations on a 2D axis.Let's consider two numbers 30y+11 and 30 x+13. Now, these two numbers will be composite for the following conditions of integer values of 'a' and 'b'.

$$(30a + 1)(30b + 11) = 30((30b + 11)a + b) + 11 = 30q1 + 11[a > 0, b >= 0]$$

$$(30a + 13)(30b + 17) = 30((30b + 17)a + 13b + 7) + 11 = 30q2 + 11[a, b >= 0]$$

$$(30a + 23)(30b + 7) = 30((30b + 7)a + 23b + 5) + 11 = 30q3 + 11[a, b >= 0]$$

$$(30a + 19)(30b - 1) = 30((30b - 1)a + 19b - 1) + 11 = 30q4 + 11[a >= 0, b > 0]$$

Let's call the set of equations q1,q2,q3,q4 as f2.Thus 30Y +11 will be composite when Y acquires integers produced by f2 and will be a prime number when it takes up whole numbers that cannot be generated by f2 . q1,q2,q3,q4 represents families of straight lines with slopes and intercepts as functions of b.

$$(30a + 1)(30b + 13) = 30((30b + 13)a + b) + 13 = 30t1 + 13[a > 0, b >= 0]$$

$$(30a + 11)(30b + 23) = 30((30b + 23)a + 11b + 8) + 13 = 30t2 + 13[a, b >= 0]$$

$$(30a + 19)(30b + 7) = 30((30b + 7)a + 19b + 4) + 13 = 30t3 + 13[a, b >= 0]$$

$$(30a + 17)(30b - 1) = 30((30b - 1)a + 17b - 1) + 13 = 30t4 + 13[a >= 0, b > 0]$$

Let's call set of equations t1,t2,t3,t4 as f3. Thus 30X +13 will be composite when X acquires values produced by f3 and will be a prime number when it takes up whole numbers that cannot be generated by f3. Hence, 30k+11 and 30k+13 will be a twin prime pair when k lies outside the range of both f2 and f3. Technique used to prove the conjecture: Now, let's consider a domain J= [30n,60n] where n belongs to the set of positive integers and project the family of straight lines f2 and f3 as described above such that the integer values of y achieved by f2 and f3 (i.e. for given conditions of 'a' and 'b')in this domain will give composite numbers of the form 30y +11 or 30 y+13 and the values of y which can't be generated by both f2 and f3 will produce twin primes of the form 30y+11 and 30y+13.Now, if we can show that for every value of n, there will always be an integer value of y in the domain J that cannot be produced by f2 and f3, we can say that there will be infinite values of y such that 30y+11 and 30y+13 form a twin prime pair(as there are infinite values of n).Similarly, if we can find a set of straight lines(say 'd') which will always produce more number of integer points in domain J than f2 and f3 and show that for every value of n, there will always be an integer value of y in the domain J that cannot be produced by 'd' , we can say that there will always be an integer value of y in the domain J for every value of n that

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cannot be achieved by f2 and f3 as the set of straight lines 'd' achieves more number of points in J than f2 and f3.Thus, there will be infinite values of y such that 30y+11 and 30y+13 form a twin prime pair(as there are infinite values of n).Thus, the twin prime conjecture is proved.

Keywords *– 11A41,primes,twin prime conjecture,polignac's conjecture*

1 Introduction

I)GENERATING COMPOSITES OF THE FORM 30Y+11: a) Multiplying numbers of the form 30b+11 and 30a+1

$$(30a + 1)(30b + 11) = 30((30b + 11)a + b) + 11 = 30q1 + 11; [a > 0, b >= 0]$$

b) Multiplying numbers of the form 30a+13 and 30b+17

$$(30a + 13)(30b + 17) = 30((30b + 17)a + 13b + 7) + 11 = 30q2 + 11; [a, b >= 0]$$

c) Multiplying numbers of the form 30a+23 and 30 b+7

$$(30a + 23)(30b + 7) = 30((30b + 7)a + 23b + 5) + 11 = 30q3 + 11; [a, b >= 0]$$

d) Multiplying numbers of the form 30a+19 and 30b-1

$$(30a + 19)(30b - 1) = 30((30b - 1)a + 19b - 1) + 11 = 30q4 + 11; [a >= 0, b > 0]$$

Let's call the set of equations q1,q2,q3,q4 as f2.Thus 30Y +11 will be composite when Y acquires integers produced by f2 and will be a prime number when it takes up whole numbers that cannot be generated by f2 . Note: q1,q2,q3,q4 represents families of straight lines with slopes and intercepts as functions of b. Thus, as b increases both slope and intercept of the individual lines increases.

II)GENERATING COMPOSITES OF THE FORM 30X+13: a) Multiplying numbers of the form 30a+1 and 30b+13

$$(30a + 1)(30b + 13) = 30((30b + 13)a + b) + 13 = 30t1 + 13; [a > 0, b >= 0]$$

b) Multiplying numbers of the form 30a+11 and 30b+23

$$(30a + 11)(30b + 23) = 30((30b + 23)a + 11b + 8) + 13 = 30t2 + 13; [a, b >= 0]$$

c) Multiplying numbers of the form 30a+19 and 30 b+7

$$(30a + 19)(30b + 7) = 30((30b + 7)a + 19b + 4) + 13 = 30t3 + 13; [a, b >= 0]$$

d) Multiplying numbers of the form 30a+19 and 30b-1

$$(30a + 17)(30b - 1) = 30((30b - 1)a + 17b - 1) + 13 = 30t4 + 13; [a >= 0, b > 0]$$

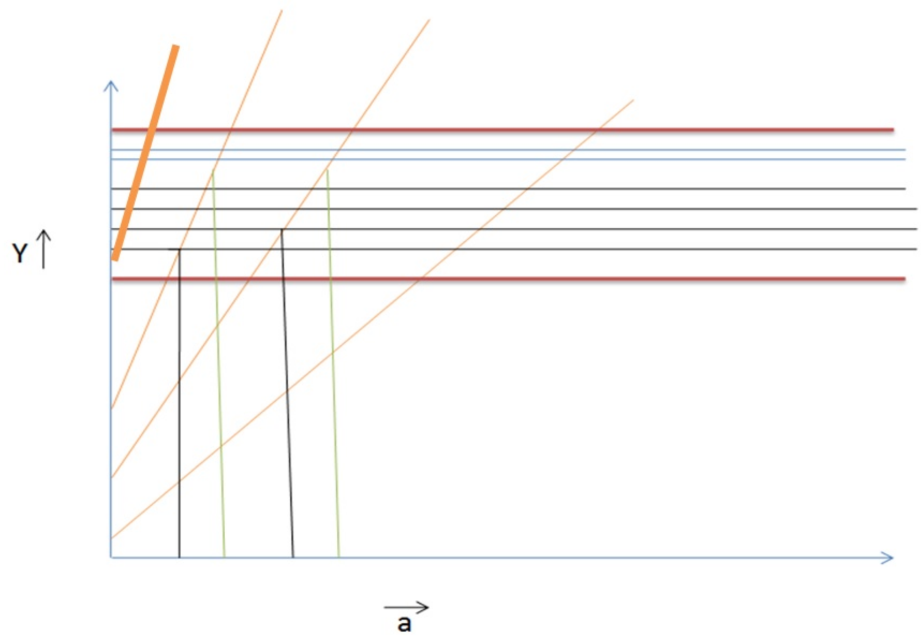
Let's call set of equations t1,t2,t3,t4 as f3. Thus 30X +13 will be composite when X acquires values produced by f3 and will be a prime number when it takes up whole numbers that cannot be generated by f3.Hence, 30y+11 and 30y+13 will be twin prime pair when y lies outside the range of both f2 and f3.

2 Discussion and Results

: Let's consider the equation q1=30ab + 11a + b=a(30b+11)+b. It's easy to observe that q1 represents a family of straight lines for natural values of b. The straight lines are given by y1=41a+1,y2=71a+2,y3=101a+3 and so on. Now, we are interested in projecting the set of equations given by f2 and f3 as described below on the a-y axis.f2 and f3 are given by: [a and b are integers]

$$y = (30b + 11)a + b; [a > 0, b >= 0] : L1$$

$$y = (30b + 17)a + 13b + 7; [a, b >= 0] : L2$$
$$y = (30b + 7)a + 23b + 5; [a, b >= 0] : L3$$
$$y = (30b - 1)a + 19b - 1; [a >= 0, b > 0] : L4$$
$$y = (30b + 13)a + b; [a > 0, b >= 0] : L5$$
$$y = (30b + 23)a + 11b + 8; [a, b >= 0] : L6$$
$$y = (30b + 7)a + 19b + 4; [a, b >= 0] : L7$$
$$y = (30b - 1)a + 17b - 1; [a >= 0, b > 0] : L8$$



Explanation of the graph: Vertical axis represents the y axis and the horizontal axis represents the axis along which Vertical axis represents the y axis and horizontal axis represents the axis along which 'a' varies.The horizontal black and blue lines represent integer values of y. The bold red horizontal lines represent the lower and upper limit of the selected domain [30n,60n] (explained later).The inclined orange lines represent the families of straight lines projected on the domain. The vertical black lines represent the integer values of 'a' for which the straight lines produce integer values of y in the selected domain. The horizontal blue lines inside the selected domain represent the unachievable points i.e. for no integer values of a, will the straight lines be able to achieve these integer values of y in the selected domain. These unachievable points in the selected domain results in the formation of prime numbers(explained later).

Target: Now, let's consider a domain J= [30n,60n] where n belongs to the set of positive integers and project the family of straight lines f2 and f3 as described above such that the integer values of y achieved by f2 and f3 (i.e. for given conditions of 'a' and 'b')in this domain will give composite numbers of the form 30y +11 or 30 y+13 and the values of y which can't be generated by both f2 and f3 will produce twin primes of the form 30y+11 and 30y+13 (see the horizontal blue lines inside the domain in fig. above).Now, if we can show that for every value of n, there will always be an integer value of y in the domain J that cannot be produced by f2 and f3, we can say that there will be infinite values of y such that 30y+11 and 30y+13 form a twin prime pair(as there are infinite values of n).Similarly, if we can find a set of straight lines(say 'd') which will always produce more number of integer points in domain J than f2 and f3 and show that for every value of n, there will always be an integer value of y in the domain J that cannot be produced by 'd' , we can say that there will always be an integer value of y in the domain J for every value of n that cannot be achieved by f2 and f3 as the set of straight lines 'd' achieves more number of points in J than f2 and f3.Thus, there will be infinite values of y such that 30y+11 and 30y+13 form a twin prime pair(as there are infinite values of n).

No. of integer values of y that a straight line y= ma +c will produce for integer values of a, m and c such that y is less than n is equal to the greatest integer less than or equal to [(n-c)/m]. Hence, to maximise the number of points we can put y-intercept c=0.We also observe that by reducing the slope of the equation (i.e. m) we can get more number of integer values of y such that y is less than n. Let's call the set of following equations as d:

$$y = (30b + 11)a; [a > 0, b >= 0] : d1$$

$$y = (30b + 17)a; [a, b >= 0] : d2$$

$$y = (30b + 7)a; [a, b >= 0] : d3$$

$$y = (30b - 1)a; [a >= 0, b > 0] : d4$$

$$y = (30b + 13)a; [a > 0, b >= 0] : d5$$

$$y = (30b + 23)a; [a, b >= 0] : d6$$

$$y = (24b + 7)a; [a, b >= 0] : d7$$

$$y = (24b - 1)a; [a >= 0, b > 0] : d8$$

d1 to d6 represent L1 to L6 with y-intercept being 0 and d7 and d8 represent L7 and L8 with y-intercept as 0 and slope of d7 and d8 being less than L7 and L8.Thus, the set of equations given by d will achieve more number of integer points in the domain J than the set of equations given by f2 and f3 (as y-intercept has been reduced to zero and the slopes of the last 2 lines have been reduced as well) .By setting y-intercept to zero, we ensure that lines(belonging to f2 and f3) with intercepts greater than 30n but less than 60n (see the bold orange line in the figure that is not able to cover the whole domain J) will be passing through the whole domain J thus producing more integer points in J than the previous case. Now, our target is to show that for infinte values of n, the set of equations given by 'd' will never be able to produce every integer point in the domain J i.e. if we can show that for every such value of n, there will always be an integer value of y in the domain J that cannot be produced by 'd', we can say that there will be infinite values of y such that 30y+11 and 30y+13 form a twin prime pair. Let's take a number $k = 2^p * 3^q$ where p and q are non negative integers such that $30n < k < 60n$ and $2 * k > 60n$. Now, let's take a line D : y=ka and project it on the a-y axis along with the set of lines given by 'd'.

3 Conclusion

As,$30n < k < 60n$ and $2 * k > 60n$, therefore the line D will only have one point in the domain J (i.e. y=k*1).As , k is co-prime with the slopes of the lines given by 'd' so, the lines given by 'd' will never be able to achieve the value of y=k in the domain J for integer values of 'a' and 'b'. Thus ,there will always be a value of y in the domain J that cannot be generated by the lines given by 'd'. It has already been shown that the number of points achieved by the set of equations given by 'd' is more than the number of integer values of y generated by f2 and f3 in the selected domain J. Thus, we can say that f2 and f3 will always fail to generate at least one value of y in the domain J.There can be infinite such domains J in which there will be infinite such 'k'. Hence, we can say that there will be infinite number of twin primes of the form 30y+11 and 30y+13 as there are infinite such values of y for infinite values of integer n.Thus,the twin prime conjecture is proved.

References

[1] Arenstorf, R. F., There Are Infinitely Many Prime Twins, *Preprint. 26*, May 2004.