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Identities for Near and Deficient Hyperperfect Numbers

Bhabesh Das[1,*] <mtbdas99@gmail.com>
Helen K.Saikia[2] <hsaikia@yahoo.com>

[1]Department of Mathematics,B.P.C.College,781127,Assam,India
[2]Department of Mathematics,Gauhati University,781014,Assam,India

Abstaract – It is well-known that a positive integer n is said to be $k-$hyperperfect number if

$$\sigma(n) = \frac{k+1}{k}n + \frac{k-1}{k},$$

where k is a positive integer and $\sigma(n)$ is the sum of all positive divisors of n.
We call a number n is near $k-$hyperperfect number if

$$\sigma(n) = \frac{k+1}{k}n + \frac{k-1}{k} + d$$

and deficient $k-$hyperperfect number if

$$\sigma(n) = \frac{k+1}{k}n + \frac{k-1}{k} - d,$$

where d is a proper divisor of n. In this paper, for any prime number q, we present two classes of near $(q-1)-$hyperperfect number and one class of deficient $(q-1)-$hyperperfect number with two distinct prime factors and also present some numerical results.

Keywords – *Perfect number, Hyperperfect number, Near perfect number, Deficient perfect number.*

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[*]*Corresponding Author.*

1 Introduction

For any positive integer n, divisor function $\sigma(n)$ denote the sum of all positive divisors of n,

$$\sigma(n) = \sum_{d|n} d.$$

Definition 1.1. A positive integer n is called perfect number, if $\sigma(n) = 2n$. All known perfect numbers are even. The Euclid-Euler theorem gives the general form of even perfect numbers.

Theorem 1.2. ([3],[4])(Euclid-Euler Theorem) An even integer n is perfect if and only if there exists prime number p such that $n = 2^{p-1}M_p$, where $M_p = 2^p - 1$ is odd prime number.

The prime of the form $M_p = 2^p - 1$ is called Mersenne prime. There is a one-to-one relationship between even perfect numbers and Mersenne primes. Each Mersenne prime generates one even perfect number. Only 49 even perfect numbers are found so far (March,2016[?]). Probably, there are infinitely many even perfect numbers. To search for new ones is keep on going by the distributed computing project GIMPS (Great Internet Mersenne Prime). On the other hand, no odd perfect number is found, although numbers up to 10^{1500} have been checked [?]. The existence of odd perfect number is one of the most difficult open problem of mathematics.

One possible generalization of perfect numbers is the hyperperfect numbers. D.Minoli and R.Bear [6] introduced the notion of hyperperfect numbers.

Definition 1.3. A natural number n is called $k-$hyperperfect number if

$$n = 1 + k[\sigma(n) - n - 1]$$

This equation can also be written as

$$\sigma(n) = \frac{k+1}{k}n + \frac{k-1}{k}$$

It is widely believed that there are $k-$hyperperfect numbers for every natural number k. If n is $1-$hyperperfect number, then $\sigma(n) = 2n$ i.e., $1-$hyperperfect numbers are the perfect numbers. In 2000 J.S.Craine [2] computed all hyperperfect numbers less than 10^{11} and obtained 2190 hyperperfect numbers for 1932 different values of k and conjectured the following.

Conjecture 1.4. For any odd integer $k > 1$, all $k-$hyperperfect numbers are of the form $n = p^2q$, where $p = \frac{3k+1}{2}$ and $q = 3k + 4$ are prime numbers.

If n is a $2-$hyperperfect number, then n is a solution of the equation

$$\sigma(n) = \frac{3}{2}n + \frac{1}{2}.$$

For $2-$hyperperfect numbers, there is the following conjecture in [1].

Conjecture 1.5. All $2-$hyperperfect numbers are of the form $n = 3^{l-1}(3^l - 2)$, where $3^l - 2$ is prime.

Moreover, for any prime of the form $q = k + 1$,where $k \geq 1$, J.C.M.Nash [7] obtained the following form of a $k-$hyperperfect number with exactly two distinct prime divisors.

Theorem 1.6. If $k \geq 1$ and $q = k + 1$ is prime, then $n = q^{k-1}p$ is $k-$hyperperfect number, where $p = q^k - (q - 1)$ is a prime.

For more results on hyperperfect numbers see [1],[?],[?].

Definition 1.7. If n is a near perfect number, then

$$\sigma(n) = 2n + d,$$

and if n is deficient perfect number, then

$$\sigma(n) = 2n - d$$

where d is a proper divisor (other than 1 and n) of n , known as the redundant divisor of n.

There are infinitely many near perfect numbers. P. Pollack and V. Shevelev [8] characterized the even near perfect numbers with exactly two distinct prime factors and obtained following two results.

Proposition 1.8. If $m = 2^{p-1}M_p$ is a perfect number, where $M_p = 2^p - 1$ is Mersenne prime, then $n_1 = 2m$, $n_2 = 2^p m$ and $n_3 = (2^p - 1)m$ are near-perfect numbers with redundant divisors 2^p, $2^p(2^p - 1)$ and $2^p - 1$ respectively.

Proposition 1.9. Let l and t be two non-negative integer with $l \geq t$. If $(2^{l+1} - 2^t - 1)$ is an odd prime, then $n = 2^l(2^{l+1} - 2^t - 1)$ is a near perfect number with redundant divisor 2^t.

$40 = 2^3.5$ is also a near perfect number with two distinct prime divisors, but it is not any one of the form given in above two propositions. In [9] Ren and Chen improved the above two popositions by proving following result.

Proposition 1.10. Let p be an odd prime and $\alpha \geq 1, \beta \geq 1$. All near perfect numbers with two distinct prime divisors are of the form $n = 2^\alpha p^\beta$ and $n = 2^\alpha p^\beta$ is near perfect number if and only if $n = 40$ or n is any one of the form given by Propositions 1.8-1.9.

M.Tang,X.Z.Ren and M.Li [?] calculated all deficient perfect numbers with at most two distinct prime factors. For more results on near perfect and deficient perfect numbers see [5],[8],[9],[?].

In this article, using the notion of near perfect and deficient perfect numbers, we define near $k-$hyperperfect and deficient $k-$ hyperperfect numbers and also present numerical results for some $k = q - 1$, where q is a prime.

2 Main Results

2.1 Near Hyperprfect Number

Definition 2.1. We say that a positive integer n is a near $k-$hyperperfect number with the proper divisor d (the divisor d is termed as redundant divisor of n) , if

$$\sigma(n) = \frac{k+1}{k}n + \frac{k-1}{k} + d$$

For any prime q, if we put $k = q - 1$, then for near $(q-1)-$hyperperfect numbers, we obtain

$$\sigma(n) = \frac{q}{q-1}n + \frac{q-2}{q-1} + d$$

In particular $q = 2$, near $1-$hyperperfect numbers are the solution of the equation

$$\sigma(n) = 2n + d$$

, which are near perfect numbers.

$q = 3$, near $2-$hyperperfect numbers are the solution of the equation

$$\sigma(n) = \frac{3}{2}n + \frac{1}{2} + d$$

$q = 5$, near $4-$hyperperfect numbers are the solution of the equation

$$\sigma(n) = \frac{5}{4}n + \frac{3}{4} + d$$

, etc.

It is observed that some of the near $(q-1)-$hyperperfect numbers are associated with $(q-1)-$hyperperfect numbers.

Theorem 2.2. Let l and t be two non-negative integer with $l \geq t$. Suppose that $k = q - 1$ and $n = q^l p$, where p and q are two distinct primes, then n is a near $(q-1)-$hyperperfect number with redundant divisor q^t if and only if the prime p is of the form $q^{l+1} - (q-1)q^t - (q-1)$.

Proof. **Proof.** Since $l \geq t$, so it is clear that q^t is a proper divisor of $n = q^l p$. If $n = q^l p$ is a near $(q-1)-$hyperperfect number with redundant divisor q^t, then

$$q^t = \sigma(n) - \frac{q}{q-1}n - \frac{q-2}{q-1}$$

This implies

$$\begin{aligned} q^t(q-1) &= (q-1)\sigma(n) - qn - (q-2) \\ &= (q^{l+1}-1)(p+1) - q^{l+1}p - (q-2) \\ &= q^{l+1} - p - (q-1) \end{aligned}$$

Thus $p = q^{l+1} - (q-1)q^t - (q-1)$

From the definition of near $k-$hyperperfect numbers, the sufficiency is immediate. □

Remark 2.3. For $q = 2$, we have $k = 1$ and from the Theorem 2.2,we obtain the Proposition 1.9. This theorem is the nice generalization of the Proposition 1.9.

Corollary 2.4. If $n = 3^l(3^{l+1} - 2.3^t - 2)$, where $3^{l+1} - 2.3^t - 2$ is a prime and $l \geq t$, then n is a near $2-$hyperperfect number with redundant divisor 3^t.

Corollary 2.5. If $n = 5^l(5^{l+1} - 4.5^t - 4)$, where $5^{l+1} - 4.5^t - 4$ is a prime and $l \geq t$, then n is a near $4-$hyperperfect number with redundant divisor 5^t.

Corollary 2.6. If $q = k + 1$ is a prime for some positive integer k and $p = q^{l+1} - q^2 + 1$ is an odd prime , then $n = q^l p$ is a near $k-$hyperperfect number with redundant divisor q.

Corollary 2.4, Corollary 2.5 and Corollary 2.6 are particular cases of the Theorem 2.2 corresponding to $q = 3$, $q = 5$ and $t = 1$ respectively.

Theorem 2.7. If $p = q^l - (q-1)$ is an odd prime, then $n = q^l p$ is a near $(q-1)-$hyperperfect number with redundant divisor q^l.

Proof. **Proof.** It is to be noted that if $p = q^l - (q-1)$ is an odd prime, then $q^{l-1}p$ is $(q-1)-$hyperperfect number.

For $n = q^l p$, we have $\sigma(n) = \sigma(q^l)\sigma(p) = \frac{q^{l+l}-1}{q-1}(p+1)$

$$\begin{aligned} &= \frac{q^{l+1}p + q^{l+1} - p - 1}{q-1} \\ &= \frac{qn + q^{l+1} - q^l + q - 2}{q-1} \\ &= \frac{qn}{q-1} + \frac{q-2}{q-1} + q^l \end{aligned}$$

It is obvious that q^l is a proper divisor of n. □

Theorem 2.7 is a particular case of the Theorem 2.2, corresponding to $t = l$.

Corollary 2.8. If $p = 3^l - 2$ is an odd prime, then $n = 3^l p$ is a near $2-$hyperperfect number with redundant divisor 3^l.

Corollary 2.9. If $p = 5^l - 4$ is an odd prime, then $n = 5^l p$ is a near $4-$hyperperfect number with redundant divisor 5^l.

Corollary 2.8 and Corollary 2.9 are the particular cases of the Theorem 2.7 corresponding to $p = 3$ and $p = 5$ respectively .

Following are the some numerical examples of near $(q-1)-$hyperperfect numbers of the form $q^l p$ with redundant divisor q^t,$(l \geq t)$ for different values of q:

- Near $2-$hyperfectnumber: $63, 171, 1647, 1971, 6399, 18063, 58563, 163539,$

$$172287, 529983, 1238571, 1553499, 1588491, 11155887, 13281651,$$

$$13990239, 14331411, 100429227, 119561103, 387381123, 1076128659,$$

$$1161867807, 9685394127, 10374141663, 10450669167, 10457046459,$$

$$10459172223, 93884544207, 847258848747, 846512705583,$$
$$659001188907, 7562832177123, 7618620727539, 7625508202899,$$
$$7625584730403, 68442081441291, 480412612856367,$$
$$617666394017331, 4323713687894187, 5558495621389119,$$
$$5559058155939147, 46325504463015699, 50016293645749407,$$
$$50026461109062723, 50031537867150579, 50031482078600163,$$
$$350220814918156971, 449048558323588491, 450283898142587583,$$
$$4051319803901885439, 4052554962408095679, 4052555129773746927,$$
$$4052555143720884531, \ldots$$

$$63 = 3^2.7 = 3^2(3^3 - 2.3^2 - 2)$$
$$171 = 3^2.19 = 3^2(3^3 - 2.3 - 2)$$
$$1647 = 3^3.61 = 3^3(3^4 - 2.3^2 - 2)$$
$$1971 = 3^3.73 = 3^3(3^4 - 2.3 - 2)$$

- Near $4-$hyperfect number : $2525, 65125, 75125, 1638125, 9753125, 41003125,$

$$47253125, 48753125, 1212828125, 1220328125, 6103203125, 25634453125,$$
$$758055078125, 762930078125, 640867578125, 18951408203125,$$
$$19068595703125, 10013580126953125, 11920318408203125,$$
$$11920923876953125, 11920927783203125, 250339507080078125,$$
$$296115874267578125, 7402896876220703125, 7450199122314453125,$$
$$7450565333251953125, \ldots$$

$$2525 = 5^2.101 = 5^2(5^3 - 4.5 - 4)$$
$$65125 = 5^3.521 = 5^3(5^4 - 4.5^2 - 4)$$
$$75125 = 5^3.601 = 5^3(5^4 - 4.5 - 4)$$

- Near $6-$hyperperfect number : $2107, 115591, 40238359, 1735104259,$

$$13840581307, 96646182871, 96853715707, 4664508845479, 4735692608227,$$
$$4747314447043, 1628413355788807, 11394825604814407,$$
$$11398313609189059, 558541793049995419, \ldots$$

- Near $10-$hyperperfect number : $1758251, 17863351, 4177245830955131, \ldots$
- Near $12-$hyperperfect number : $26533, 342901, 62379421, 815387989,$

$$9851174437, 10546234933, 137854036333, 1792098016813,$$
$$23298027200773, \ldots$$

- Near $16-$hyperperfect number : $408923729, 2862421147538373073,$

$$2862422933332277969, \ldots$$

- Near $18-$hyperperfect number : $46922419, 16981217263, 306595660699,$

$$321838526143, 6131021688019, 39956157955263259, \ldots$$

- Near $22-$hyperperfect number : $1726240345127, 951086774638199, \ldots$
- Near $28-$hyperperfect number : $19804709, \ldots$
- Near $30-$hyperperfect number : $27484014751, 886609951, 819627428106271,$

$$25382889306407071, 24417545445441508831, \ldots$$

- Near $36-$ hyperperfect number : $92433671173, 129959175942829, \ldots$
- Near $40-$hyperperfect number : $4747347401, 13422654675904361,$

$$537233749706241961, 550328837077830761, ..$$

- Near $42-$hyperperfect number : $143591491, 265640917207, 271671682171,$

$$11688056687959, 491176086680707, 929293467799620043, \ldots$$

- Near $46-$hyperperfect number : $496068474283, 52599121685959727,$

$$2472158708690236847, \ldots$$

Theorem 2.10. Suppose that $p = q^l - (q-1)$ is an odd prime, then $n = q^{l-1}p^2$ is a near $(q-1)-$hyperperfect number with redundant divisor p.

Proof. **Proof.**For $n = q^{l-1}p^2$, we get

$$\begin{aligned}\sigma(n) &= \sigma(q^{l-1})\sigma(p^2)\\ &= \tfrac{q^l-1}{q-1}(p^2+p+1)\\ &= \tfrac{q^lp^2+q^l(p+1)-(p^2+p+1)}{q-1}\\ &= \tfrac{qn+(q^l-p)(p+1)-1}{q-1}\\ &= \tfrac{qn+(q-1)(q^l-q+2)-1}{q-1}\\ &= \tfrac{qn}{q-1}+(q^l-q+2)-\tfrac{1}{q-1}\\ &= \tfrac{qn}{q-1}+\tfrac{q-2}{q-1}+q^l-(q-1)\\ &= \tfrac{qn}{q-1}+\tfrac{q-2}{q-1}+p\end{aligned}$$

□

Corollary 2.11. Suppose that 3^l-2 is an odd prime, then $n = 3^{l-1}(3^l-2)^2$ is a near $2-$hyperperfect number with redundant divisor 3^l-2.

Corollary 2.12. Suppose that 5^l-4 is an odd prime, then $n = 5^{l-1}(5^l-4)^2$ is a near $4-$hyperperfect number with redundant divisor 5^l-4.

Following are the numerical examples of near $(q-1)-$ hyperperfect numbers of the form q^lp^2 with redundant divisor p for different values of q:

- Near $2-$hyperperfect numbers with redundant divisor 3^l-2 :

$$147, 168507, 4704561, 128432547, 2541349293921, \ldots$$

- Near $4-$hyperperfect number with redundant divisor 5^l-4 :

$$6087900625, ..$$

- Near $6-$hyperperfect number with redundant divisor 7^l-6 :

$$12943, 5564881, ..$$

- Near $10-$hyperperfect number with redundant divisor 11^l-10 :

$$211149961, \ldots$$

- Near 12−hyperperfect number with redundant divisor $13^l - 12$:

$$320437, 1790654745997, 3937121881827121, 8650372907585347237, ...$$

Remark 2.13. From Corollary 2.4, Corollary 2.8 and Corollary 2.11 we obtain odd near 2−hyperperfect numbers with two distinct prime divisors. Moreover $627 = 3.11.19, 663 = 3.13.17$ are the examples of near 2− hyperperfect numbers with three distinct prime divisors.

Conjecture 2.14. All near 2−hyperperfect numbers with two distinct prime divisors are any one of the form given by Corollary 2.4,Corollary 2.8 and Corollary 2.11.

Conjecture 2.15. For every prime q, there exist near $(q-1)-$hyperperfect numbers.

Open problem : Does near $k-$hyperperfect number exist for every natural number k?

2.2 Deficient Hyperperfect Number

Definition 2.16. We say that a positive integer n is deficient $k-$hyperperfect number with redundant divisor d if

$$\sigma(n) = \frac{k+1}{k}n + \frac{k-1}{k} - d$$

For any prime q, if we put $k = q-1$, then for deficient $(q-1)-$hyperperfect numbers, we obtain

$$\sigma(n) = \frac{q}{q-1}n + \frac{q-2}{q-1} - d$$

Inparticular $q = 2$, deficient 1−hyperperfect numbers are the solution of the equation

$$\sigma(n) = 2n - d$$

, which are deficient perfect numbers.

$q = 3$, deficient 2−hyperperfect numbers are the solution of the equation

$$\sigma(n) = \frac{3}{2}n + \frac{1}{2} - d$$

$q = 5$, deficient 4−hyperperfect numbers are the solution of the equation

$$\sigma(n) = \frac{5}{4}n + \frac{3}{4} - d$$

, etc.

Theorem 2.17. If $k = q-1$ and n is of the form $q^l p$, where $p = q^{l+1} + (q-1)q^t - (q-1)$ and q are two distinct primes and $l \geq t$, then n is a deficient $(q-1)-$hyperperfect number with redundant divisor q^t.

Proof. **Proof.** For $n = q^l p$, we have $\sigma(n) = \sigma(q^l)\sigma(p) = \frac{q^{l+1}-1}{q-1}(p+1)$

$$= \frac{q^{l+1}p+q^{l+1}-p-1}{q-1}$$

$$= \frac{qn+q^{l+1}-q^{l+1}-(q-1)q^t+(q-1)-1}{q-1}$$

$$= \frac{qn}{q-1} + \frac{q-2}{q-1} - q^t$$

Since $l \geq t$, so q^t is a proper divisor of n. □

Corollary 2.18. If $n = 3^l(3^{l+1} + 2.3^t - 2)$, where $3^{l+1} + 2.3^t - 2$ is prime and $l \geq t$, then n is a deficient 2−hyperperfect number with redundant divisor 3^t.

Corollary 2.19. If $n = 5^l(5^{l+1} + 4.5^t - 4)$, where $5^l + 4.5^t - 4$ is prime and $l \geq t$, then n is a deficient 4−hyperperfect number with redundant divisor 5^t.

For $t = 1$, from the Theorem 2.17, we obtain the following Corollary

Corollary 2.20. If $q = k + 1$ is a prime for some positive integer k and $p = q^{l+1} + q(q-2) + 1$ is an odd prime , then $n = q^l p$ is a deficient $k-$ hyperperfect number with redundant divisor q.

Theorem 2.21. If p and q are distinct odd primes such that $k(p + 2q) = pq - 1$ for some positive integer k, then $n = pq$ is deficient $k-$hyperperfect number with redundant divisor q.

Proof. **Proof.**If $n = pq$, then $\sigma(n) = n + p + q + 1 = n + \frac{n-qk-1}{k} + 1$

$$= \frac{nk+n+k-1-qk}{k}$$

$$= \frac{(k+1)n}{k} + \frac{k-1}{k} - q$$ □

Following are the some numerical examples of deficient $(q-1)-$hyperperfect numbers of the form $q^l p$ for different q values:

- deficient 2−hyperperfect number : $35, 39, 279, 387, 2619, 178119, 294759,$

$$1605987, 1632231, 1710963, 1947159, 17533179, 14383899, 129166407,$$

$$129245139, 130189923, 138692979, 215220483, 1165410747, 1248315543,$$

$$1420502427, 10546328547, 94146013179, 156904943751, 847297112499,$$

$$847545826887, 1412146619523, 7646515002747, 68630453892387,$$

$$68693129918163, 69195226871907, 114383952708867, 5560755057680967,$$

$$50443327104908679, 83385908240052519, 450284469975229347,$$

$$461402026249267431, 550346995314155799, 4063673271827564379, ...$$

$$279 = 3^2.31 = 3^2(3^3 + 2.3 - 2)$$

$$387 = 3^2.43 = 3^2(3^3 + 2.3^2 - 2)$$

- deficient 4−hyperperfect number : $205, 80125, 2013125, 48878125,$

$$56628125, 1259703125, 30525078125, 30712578125, 54931328125,$$

$$787351953125, 19097892578125, 19195548828125, 492095908203125,$$

$$307559966064453125, 7450595850830078125, 7450656885986328125,$$

$$7498264307861328125, 7688999171142578125, ...$$

$$205 = 5.41 = 5.(5^2 + 4.5 - 4)$$

$$80125 = 5^3.641 = 5^3.(5^4 + 4.5 - 4)$$

- deficient 6−hyperfperfect number : $18571, 30919, 835891, 922327,$

$$40440043, 45280459, 96922893319, 98583156007, 5328890631127,$$

$$232642343358859, 11398906807211959, 558549931726869607, ...$$

- deficient 10−hyperperfect number : $306251, 19620271, 37189471,$

$$2375516891, 4501390091, 308889537011, 4180101091250051,$$

$$4522475095983251, 509244524691537371, ...$$

- deficient 12−hyperperfect number : $4069, 395629, 67177669,$

$$1792213860229, 302884837439197, 303002302663021,$$

$$51187546563010741, 51207398185836997,$$

$$98434408889500021, ...$$

- deficient $16-$hyperperfect number : $2751569, 411596401, 432977777,$
  $34383485708977, ...$
- deficient $18-$hyperperfect number : $4815379, 322729921783, 338775043303,$
  $116506304019739, 42058790993991223, ...$
- deficient $22-$hyperperfect number : $23299, 3546157319, 6661347331,$
  $952812873103939, 954532457990563, ...$
- deficient $28-$hyperperfect number : $14523781224989, 12200992187930309,$
  $10272408498835954829, ...$
- deficient $30-$hyperperfect number : $57691, 27539426011, 26440453329571,$
  $49997324646954331, ...$
- deficient $36-$hyperperfect number : $71118181, 136764469,$
  $97426436077, 187296204349, 133379219902069, ...$
- deficient $42-$hyperperfect number : $155359, 502598642701807,$
  $502857965595259, 993496880006443, ...$
- deficient $46-$hyperperfect number : $453708719, 1002462249739, ...$

Conjecture 2.22. For every prime q ,there exist deficient $(q-1)-$hyperperfect numbers.

Open problem : Does deficient $k-$hyperperfect number exist for every natural number k?

3 Conclusion

we obtained some certain forms of near and deficient hyperperfect numbers for $k = q - 1$, where q is a prime. There is also a scope for the study of these two equations $\sigma(n) = \frac{k+1}{k}n + \frac{k-1}{k} + d$ and $\sigma(n) = \frac{k+1}{k}n + \frac{k-1}{k} - d$ for other values of k.

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